CERTIFIED PUBLIC ACCOUNTANT

FOUNDATION LEVEL 1 EXAMINATION

F1.1: BUSINESS MATHEMATICS AND QUANTITATIVE METHODS

DATE: THURSDAY 28, NOVEMBER 2024

MARKING GUIDE & MODEL ANSWERS

QUESTION ONE

Marking guide

	Marks
Calculation of moving total (0.5 Marks each * 8)	4
Computation of cumulative totals (0.5 Marks each * 8)	4
Lines drawn on Z chart (1 Mark each, maximum 3)	3
Title of the graph	1
Axes (0.5 Marks each, Maximum 1)	1
Maximum marks	**13**
Calculation of the range	**1**
Formula of variance	0.5
Calculation of variance	1.5
Maximum marks	**2**
Formula of coefficient variation	0.5
Calculation of coefficient variation	0.5
Maximum marks	**1**
Formula of coefficient skewness	1
Calculation of coefficient skewness	2
Maximum marks	**3**
Total marks	**20**

Model Answer

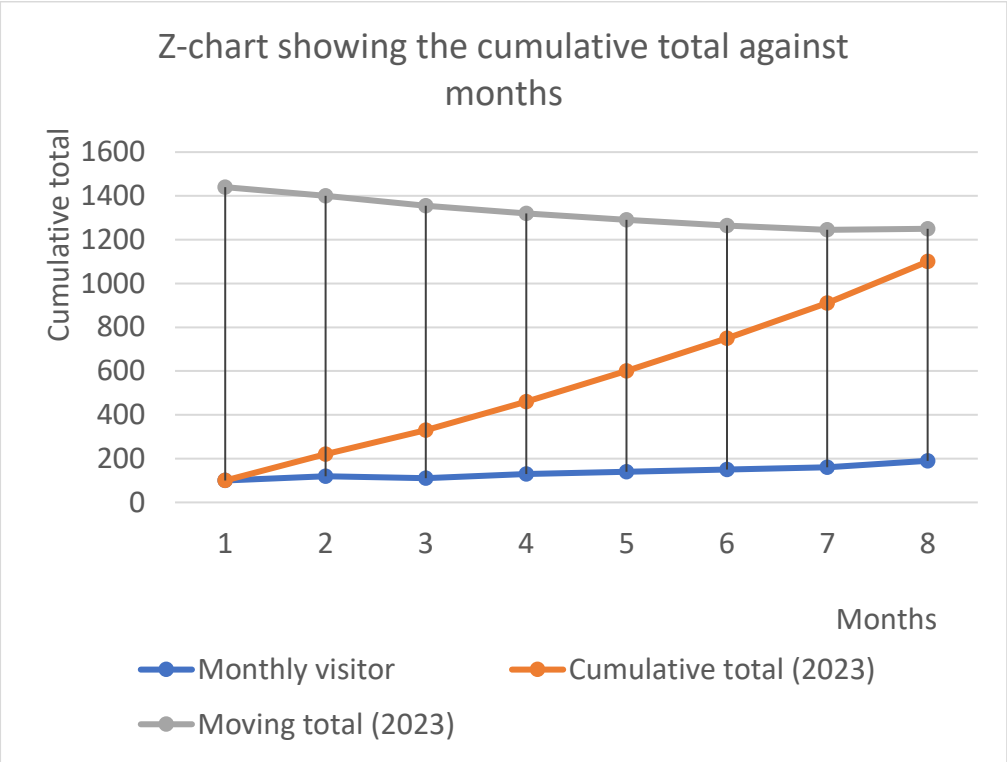
i) Find the moving totals.

Monthly visitors (2022)	**Monthly visitors (2023)**	**Moving total (2023)**	**Cumulative total (2023)**
150	100	1440	100
160	120	1400	220
155	110	1355	330
165	130	1320	460
170	140	1290	600
175	150	1265	750
180	160	1245	910
185	190	1250	1100
1340			

The first moving total is obtained from the sum of monthly visitors of 2022 (1,340) and then we add the first monthly visitor of 2023 (100) which gives 1,440.

The second moving total is obtained by the previous moving total (1,440) minus the second monthly visitor of 2022 (160) plus the second monthly visitor of 2023 (120) which gives (1,400). The rest are done in the same way.

ii) Z-chart



b)

i) The range is the difference between the maximum and minimum sales figures.

Maximum Sales: 250

Minimum Sales: 150

Range=Max−Min=250−150=100

ii) Calculate the Variance

Variance measures how much the sales figures vary from the mean.

$$Mean = \frac{\sum x}{n} = \frac{150+200+180+220+250+210+190}{7} = 200$$

$$Var(x) = \frac{\sum(x - Mean)^2}{n}$$

$$= \frac{(150-200)^2 + (200-200)^2 + (180-200)^2 + (220-200)^2 + (250-200)^2 + (210-200)^2 + (1}{7}$$

$$= \frac{6000}{7} = 857.14$$

iii) $CV = \frac{\delta}{mean} \times 100 = \frac{\sqrt{857.14}}{200} \times 100 = \frac{29.28}{200} \times 100 = 14.64\%$

iv) $Skewness = \frac{n}{(n-1)(n-2)} \times \frac{\sum(x-mean)^3}{\delta^3}$

$$\sum(x - mean)^3 = -125000 + 0 - 800 + 800 + 125000 + 1000 - 1000 = 0$$

Therefore $Skewness = \frac{n}{(n-1)(n-2)} \times \frac{\sum(x-mean)^3}{n} = \frac{7}{(7-1)(7-2)} \times \frac{0}{26.68^3} = 0$

QUESTION TWO

Marking guide

		Marks
a)	Functions of statistics (1 mark for each)	**4**
b)	State steps for random sampling (1 mark to each correct step)	**6**
c)	Formula of Z score	1
	Computation of z-score	1
	Probability value	1
	Interpretation	1
	Maximum marks	**4**
d) i)	Computation of probability of exactly 2	2
ii)	Computation of probability of 0	1
	Computation of probability of 1	1
	Total probability	1
iii)	Probability of at least	1
	Maximum marks	**6**
	Total marks	**20**

Model Answer

a) Main functions of statistics are:
1. Presenting data in a definite form which makes the statement logical and convincing. Numerous figures can be summarized into a single intelligible figure.
2. Statistics reduces the complexity of data. Usually, raw data is unintelligible. Ling different statistical measures such as averages, graphs, dispersions, etc. The unintelligible data can be made simple and can easily be interpreted and conclusions drawn.

3. Facilitating comparison. Comparison wouldn't be possible without the use of the various statistical measures. Both graphical and numerical measures provide ample scope for comparison.
4. Establishing trends and tendencies. After studying data over a period of time, a trend can be established which can help in forecasting. Planners can forecast the future produce in agriculture, future population by considering present day figures.
5. Drawing valid conclusions/inferences. Statistical measures can be used draw conclusions from a given survey which can be used to evaluate different projects.
6. Testing hypotheses. Statistics can be used to test the truth of new ideas. This helps in developing new theories.

b) Possible Steps to Ensure Proper Random Sampling:
Define the Population: Identify the entire group of customers visiting the coffee shop within the week.
Select a Sampling Frame: Create a list of all customers who visit the shop during that week. This could involve tracking customer receipts or using a digital loyalty program.
Determine Sample Size: Decide on the sample size (in this case, 50 customers) that is representative of the population.
Use Random Selection: Implement a random selection method, such as: Using a random number generator to select customers from the list. Drawing names or numbers from a hat if a list is available.
Conduct the Survey: Reach out to the selected customers and conduct the survey, ensuring that participation is voluntary and anonymous.
Analyze the Data: Collect and analyze the feedback to draw conclusions about customer satisfaction.

c) Given: $\mu = 75$

And $\sigma = 10$

$\boldsymbol{Z} = \frac{x-\mu}{\sigma} = \frac{85-75}{10} = \frac{10}{10} = 1.00$

Look up the Z-score of 1.0 in the standard normal distribution table (Z-table).

The Z-table provides the area to the left of the Z-score.

Finding the Area:

From the Z-table, the area corresponding to Z=1.0

Z=1.0 is approximately 0.8413.

Approximately 84.13% of students scored below 85 on the exam. This means that around 84.13% of students performed at or below this score.

d) i) In Poisson distribution: $p(x = k) = \frac{e^{-\lambda} \times \lambda^k}{k!}$

$x = k = 2, \lambda = 4$

$$p(x = 2) = \frac{e^{-\lambda} \times \lambda^k}{k!} = \frac{e^{-4} \times 4^2}{2!} = 0.1465$$

The probability of receiving exactly 2 page views in the next minute is approximately 14.65%.

ii) P(x≤2) = P(x=0) + P(x=1) + P(x=2)

P(x=0) = $\frac{e^{-4} \times 4^0}{0!}$ = 0.018

P(x=1) = $\frac{e^{-4} \times 4^1}{1!}$ = 0.073

P(x=2) = $\frac{e^{-4} \times 4^2}{2!} = 0.147$

Now, sum the probabilities:

$$P(X \leq 2) = 0.018316 + 0.073264 + 0.1465 = 0.23808$$

0.238 (rounded to 3 decimal places).

iii) Calculate the probability of at least 2 page views occurring per minute. (3 Marks)

To calculate the probability of **at least 2** page views, we need to find $P(X \geq 2)$. This is the complement of the probability of having fewer than 2 page views:

$$P(X \geq 2) = 1 - P(X < 2)$$

Since $P(X < 2)$ is the probability of having fewer than 2 page views (i.e., 0 or 1 page views), we can write:

$P(X < 2) = P(X = 0) + P(X = 1)$

From the earlier calculations:

$$P(X = 0) = 0.018316$$

$$P(X = 1) = 0.073264$$

So:

$$P(X < 2) = 0.018316 + 0.073264 = 0.09158$$

Thus:

$$P(X \geq 2) = 1 - 0.09158 = 0.90842$$

So, the probability of having at least 2 page views in a minute is approximately **0.908** (rounded to 3 decimal places).

QUESTION THREE

	Marking Guide	**Marks**
a)	Integrating cost	1
	Integrating revenue	1
	Formula of profit	1
	Calculation of profit function	1
	Maximum marks	**4**
b)	i) Right formula of fixed cost	1
	Formula and Calculation of break-even sales	2
	Maximum marks	**3**
c)	Finding minor and cofactor of element through the selected Row/column	3
	Computation of determinant	1
	Maximum marks	**4**
d)	Finding marginal cost	1
	Finding price/demand function in domestic market	1
	Finding revenue function in domestic market	1
	Finding marginal revenue in domestic market	1
	Finding quantity at equilibrium in domestic market	1
	Finding price/demand in foreign market	1
	Finding revenue function in foreign market	1
	Finding quantity at equilibrium in foreign market	1
	Comment	1
	Maximum marks	**9**
	Total Marks	**20**

Answer model

a) Given $MC = 4 + 0.08x$ and $MR = 12$

$$profit = R - C$$

$$R = \int MRdx = \int 12dx = 12x$$

$$C = \int MCdx = \int (4 + 0.08x)dx = 4x + 0.04x^2$$

Therefore,

$$profit = R - C = 12x - (4x + 0.04x^2) = 8x - 0.04x^2$$

b) Let R is total sales revenue in monetary terms

P, unit price, V, unit variable costs, f, fixed cost, x, sales in physical units, Vx, total variable cost,

Total cost $C = Vx + f$ and profit π, the profit.

$$\pi = TR - Tc = Sale - cost$$

$$TC = VC + FC$$

i) $TP = Sales - VC - FC = 100000 - 60000 - 20000 = 20000$

ii) At Break Even Point (BEP), $R = C$, that $\pi = 0$

$0 = CMx - f$, $x_{ble} = \frac{f}{CM}$. $\pi = x(P - V) - f$, as $R = Px$, $x = \frac{R}{P}$

$\pi = (P - V)\frac{R}{P} - f = (\frac{P-V}{P})R - f$ and therefore, Contribution margin ratio $CMR = \frac{P-V}{p}$

$$R_{bles} = \frac{FC}{CMR}$$

$CMR = \frac{P-V}{P}$ multiplying both side by x, we get $CMR = \frac{Px-Vx}{Px} = \frac{100000-60000}{100000} = \frac{4}{10} = 0.4$

Therefore

$$R_{bles} = \frac{FC}{CMR} = \frac{20000}{0.4} = 50000$$

c) Let fixe the first row and find minor and cofactor of each element through the row

Minor and cofactor of 150: $+150\begin{vmatrix} 90 & 70 \\ 100 & 90 \end{vmatrix}$

Minor and cofactor of 80: $-80\begin{vmatrix} 200 & 70 \\ 250 & 90 \end{vmatrix}$

Minor and cofactor of 60: $+60\begin{vmatrix} 200 & 90 \\ 250 & 100 \end{vmatrix}$

And therefore, $det(A) = +150\begin{vmatrix} 90 & 70 \\ 100 & 90 \end{vmatrix} - 80\begin{vmatrix} 200 & 70 \\ 250 & 90 \end{vmatrix} + 60\begin{vmatrix} 200 & 90 \\ 250 & 100 \end{vmatrix}$

$$det(A) = 150(1100) - 80(500) + 60(-2500) = 165000 - 40000 - 150000$$

$$det(A) = -25000$$

d) To maximize profits under price discrimination, the producer will set prices so that $MC = MR$ in each market. Thus, $MC = MR_1 = MR_2$.

Finding marginal cost through derivative

With $TC = 2000 + 10Q$

$MC = \frac{d(TC)}{dQ} = \frac{d(2000+10Q)}{dQ} = 10$

Finding demand function/price in the domestic market

MC will be the same at all levels of output.

In the domestic market $Q_1 = 21 - 0.1P_1$ Hence, $P_1 = 210 - 10Q_1$

Finding the revenue function in the domestic market

$\boldsymbol{TR_1} = (210 - 10Q_1)Q_1 = 210Q_1 - 10{Q_1}^2$,

Finding marginal revenue in the domestic market

$MR_1 = 210 - 20Q_1$

Finding quantity at equilibrium in the domestic market

When $\boldsymbol{MR_1 = MC}$, $210 - 20Q_1 = 10, \quad Q_1 = 10$

When $Q_1 = 10$, then $P_1 = 210 - 10(10) = 110$

In the foreign market, the quantity, $Q_2 = 50 - 0.4P_2$

Finding the price/ demand function

From $Q_2 = 50 - 0.4P_2$, we make P_2 the subject so as to get price

$P_2 = 125 - 2.5Q_2$

Finding marginal revenue in the foreign market

Hence, $TR_2 = (125 - 2.5Q_2)Q_2 = 125Q_2 = 2.5{Q_2}^2$

Thus $MR_2 = 125 - 5Q_2$

Finding quantity at equilibrium in the foreign market

When $MR_2 = MC$, $125 - 5Q_2 = 10$, and $Q_2 = 23$

When $Q_2 = 23$, $\quad P_2 = 125 - 2.5(23) = 67.5$

Comment

The discriminating producer charges a lower price in the foreign market where the demand is relatively more elastic and a higher price $\boldsymbol{P_1 = 110}$ in the domestic market where the demand is relatively less elastic.

QUESTION FOUR

Marking guide

	Marks
a) Formula	1
Arrangement for teachers	1
Arrangement for students	1
Total number of committee	1
Maximum marks	**4**
b) Formula for FPCF	1
Computation for FPCF	1
Computation of standard error	1
Computation of interval	1
Maximum marks	**4**
c) Identification of probability (0.5 Marks each, Maximum 3)	3
Formula for Bayes Theorem	1
Calculation of Bayes Theorem	2
Maximum marks	**6**
d) Computation of expected frequency (0.5 Marks each)	1.5
Formula for chi -square	1
Computation of chi -square	1
Degree of freedom	0.5
Critical value of chi - square	1
Conclusion	1
Maximum marks	**6**
Total marks	**20**

Answer model

a) Forming the Committee

- To find the number of ways to select 4 teachers from 6 and 3 students from 5, we use the combination formula: $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Selecting teacher: : $\binom{6}{4} = \frac{6!}{(6-4)!4!} = 15$

Selecting student: : $\binom{5}{3} = \frac{5!}{(5-3)!3!} = 10$

Using principal of counting the total number of committee is $\binom{6}{4} \times \binom{5}{3} = 15 \times 10 = 150$

- Once the committee of 7 members (4 teachers + 3 students) is formed, we can arrange them in a line. The number of arrangements of 7 members is given by: $n!$

$$n! = 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

b) If a given population is relatively of small size and sample size is more than 5% of the population then the standard error should be adjusted by multiplying it by the finite population correction factor (FPCF)

$FPCF = \sqrt{\frac{N-n}{n-1}}$ where N is the population size and n sample size.

$N = 1000, n = 200$ and $FPCF = \sqrt{\frac{N-n}{n-1}} = \sqrt{\frac{1000-200}{200-1}} = \sqrt{\frac{800}{199}} = 2.005$

The standard error is then $S_{\bar{x}} = \frac{s}{\sqrt{n}} \times \sqrt{\frac{N-n}{n-1}} = \frac{3000}{\sqrt{200}} \times 2.005 = \frac{6015}{14.1421} = 425.3258$

At 99% $Z_{\alpha/2} = 2.58$, therefore the interval estimation is:

$$\left[\bar{x} \pm Z_{\frac{\alpha}{2}} \times S_{\bar{x}}\right] = [15000 \pm 2.58 \times 425.3258] = [15000 \pm 1097.3406]$$

Population mean is in [13902.6594, 16097.3406]

c) Let P(B1) be the probability the email is processed by the first boy

P(B2) be the probability the email is processed by the first boy

P(B3) be the probability the email is processed by the first boy

P(E/B1) be the probability the email is processed by the first boy

P(E/B2) be the probability the email is processed by the first boy

P(E/B3) be the probability the email is processed by the first boy

Identification of probabilities

$$P(B1) = 40/100$$

$$P(B2) = 35/100$$

$$P(B3) = 25/100$$

$$P(E/B1) = 4/100$$

$$P(E/B2) = 6/100$$

$$P(E/B3 = 3/100$$

Asked P(B1/E) be the probability the email selected and found has an error was processed by the first boy.

Using Bayes 'theorem:

$$P(B1/E) = \frac{P(B1) \times P(E/B1)}{P(B1) \times P(E/B1) + P(B2) \times P(E/B2) + P(B3) \times P(E/B3)}$$

$$P(B1/E) = \frac{0.4 \times 0.04}{0.4 \times 0.04 + 0.35 \times 0.06 + 0.25 \times 0.03} = \frac{0.016}{0.016 + 0.021 + 0.0075}$$
$$= \frac{0.016}{0.0445} = 0.3596$$

Formulation of hypothesis:

Ho: there is no a change in market share after the advertising campaign

H1: there is a change in market share after the advertising campaign

Computation of expected frequencies

	A	B	C
Obtained frequency	102	82	16
Expected frequency	$\frac{45(200)}{100} = 90$	$\frac{40(200)}{100} = 80$	$\frac{15(200)}{100} = 30$

Computation of Chi – square

$$\chi_{ob}{}^2 = \sum \frac{(fo - fe)^2}{fe}$$

$$\chi_{ob}{}^2 = \frac{(102 - 90)^2}{90} + \frac{(82 - 80)^2}{80} + \frac{(16 - 30)^2}{30}$$

$$\chi_{ob}{}^2 = 8.1$$

$$\boldsymbol{df = k - 1 = 3 - 1 = 2}$$

$$\chi^2 cv(df = 2, \alpha = 0.05) = 5.9914$$

Since $\chi_{ob}{}^2 > \chi^2 cv$, Ho is rejected. There is a change in the market share after advertising. This means that market share depends on advertising.

QUESTION FIVE

Marking guide

	Marks
a) i) Formula of Laspeyres price index	1
Computation of totals (1 Mark each, Maximum 2)	2
Computation of Laspeyres price index	1
Maximum marks	**4**
ii) Definition of CPI	**2**
iii) Advantages of CPI (1 Mark each, Maximum 2)	2
Disadvantages of CPI (1 Mark each, Maximum 2)	2
Maximum marks	**4**
b) Computation of percentage	**2**
c) Formula for period	1
Formula of simple interest	1
Computation of simple interest	1
Computation of period from simple interest	1
Maximum marks	**4**
d) Importances of time series (1 Mark for stating and 1 Mark for explanation Maximum 4)	**4**
Total marks	**20**

Model Answer

a) i) **Computation of Laspeyres Price index**

	2022		2023			
	Price (FRW) Po	Quantity Qo	Price (FRW) Pt	Quantity Qt	PtQo	PoQo
Food	4	10	5	8	50	40
Medical care	6	8	9	9	72	48
Clothes	5	5	7	11	35	25
Fuel	3	12	6	8	72	36
Communication	5	7	8	5	56	35

sum	285	184

$$LI = \frac{\sum PtQo}{\sum PoQo} \times 100 = \frac{285}{184} \times 100 = 154.89$$

ii) The **Consumer Price Index** (CPI) is a statistical measure that tracks the average change over time in the prices paid by consumers for a fixed basket of goods and services

i) Advantages

- **Inflation Measurement**: CPI is a key indicator of inflation, helping governments and central banks make informed economic decisions.
- **Cost of Living Adjustment:** It helps adjust wages, pensions, and social security benefits, ensuring that they keep pace with inflation.
- **Economic Analysis:** Economists and policymakers use CPI for analyzing economic health and formulating fiscal and monetary policies.
- **Consumer Insights:** Provides insights into consumer spending patterns and preferences.
- **International Comparisons**: Allows for comparison of inflation rates between different countries, aiding in global economic assessments.

Disadvantages

- **Substitution Bias**: CPI may not accurately reflect consumer behavior, as it doesn't account for consumers switching to cheaper alternatives when prices change.
- **Quality Changes**: The index may not fully capture improvements in product quality or new products, leading to an overestimation of inflation.
- **Basket Limitations:** The fixed basket of goods and services may not represent all consumers, particularly diverse demographic groups.
- **Regional Variations**: CPI is often based on national averages, which may overlook significant regional price differences.
- **Time Lag:** CPI data is typically released with a delay, making it less useful for real-time economic analysis.

b) Computation of percentage

Percentage of girls = (120*100)/200 = 60

c) Given Present value (PV)= 10,000

Future value (FV)=30,000

Interest rate=8%=0.08

$$T = \frac{100 \times SI}{P \times R}$$

Simple interest (SI)=FV-P

Simple interest=30000-10000=20000

$$30000 = 10000(1 + 0.08n)$$

$$3 = 1 + 0.08n$$

$$n = \frac{2}{0.08} = 25\ years$$

It will take 25 years to triple his investment

d) Importance of time series

1. Identifying Seasonal Patterns

- **Sales Trends**: By analyzing historical sales data, Fashion Trends can identify peak seasons for summer and winter apparel, allowing them to stock inventory accordingly.
- Demand Forecasting: Recognizing patterns in sales can help predict future demand, ensuring that the right amount of stock is available when needed.

2. Optimizing Inventory Levels

- Just-in-Time Inventory: Time series analysis can help determine optimal reorder points and quantities, minimizing excess inventory and reducing holding costs.
- Seasonal Stock Adjustments: By understanding seasonal demand fluctuations, the retailer can adjust inventory levels preemptively to avoid stock outs or overstocking.

3. Enhanced Marketing Strategies

- Targeted Promotions: Insights from sales data can guide promotional campaigns tailored to specific times of the year, enhancing customer engagement and driving sales.
- Timing of Advertising: Fashion Trends can schedule marketing efforts to coincide with peak shopping periods, maximizing visibility and sales impact.

4. Trend Analysis

- Long-term Trends: Analyzing long-term trends can reveal shifts in consumer preferences, enabling Fashion Trends to adapt their product offerings and marketing strategies to align with evolving market demands.

5. Performance Evaluation

- Sales Performance Metrics: Regular analysis can help evaluate the effectiveness of past marketing campaigns and inventory decisions, allowing for data-driven adjustments in strategy.
- Comparative Analysis: Comparing current year data against previous years can highlight successes and areas needing improvement, facilitating strategic planning.

6. Risk Management

- Demand Variability: Understanding fluctuations in demand can help mitigate risks associated with unsold inventory and financial losses.
- Economic Indicators: Time series analysis can incorporate external economic indicators to forecast demand, helping the retailer prepare for economic downturns or upswings.

QUESTION SIX

		Marking guide	Marks
a	i.	Computation of both means (1 mark on each variable)	2
		Computation of both variances (1 mark on each variable)	2
		Computation of covariance	1
		Computation of both parameter a and b	2
	ii.	Regression line	1
	iii	Prediction	2
		Maximum marks	**10**
b		Computation of EMV (2 Marks for each max 6)	6
		Conclusion	2
		Formula and computation of EVPI (1 Mark for formula and 1 Mark for computation)	2
		Maximum marks	**10**
		Total marks	**20**

Model Answer

	x	y	x^2	y^2	xy
	4	13	16	169	52
	17	47	289	2209	799
	3	24	9	576	72
	21	41	441	1681	861
	10	29	100	841	290
	8	33	64	1089	264
	4	28	16	784	112
	9	38	81	1444	342
	13	46	169	2116	598
	12	32	144	1024	384
	2	14	4	196	28
	6	22	36	484	132
	15	26	225	676	390
	8	21	64	441	168
	19	50	361	2500	950
sum	**151**	**464**	**2019**	**16230**	**5442**

Computation of mean

Mean of x $\bar{x} = \frac{\sum x_i}{n} = \frac{151}{15}$

Mean of y $\bar{y} = \frac{\sum y_i}{n} = \frac{464}{15}$

Computation of variance

$var(x) = \frac{\sum x^2}{n} - \bar{x}^2 = \frac{2019}{15} - \left(\frac{151}{15}\right)^2 = \frac{30285-22801}{225} = \frac{7484}{225}$

$var(y) = \frac{\sum y^2}{n} - \bar{y}^2 = \frac{16230}{15} - \left(\frac{464}{15}\right)^2 = \frac{243450-215296}{225} = \frac{28154}{225}$

Computation of covariance

$cov(x,y) = \frac{\sum xy}{n} - \bar{x}\bar{y} = \frac{5442}{15} - \frac{151}{15} \times \frac{464}{15} = \frac{81630-70064}{225} = \frac{11566}{225}$

Computation of correlation coefficient

$$\rho = \frac{cov(x,y)}{\sqrt{var(x)var(y)}} = \frac{\frac{11566}{225}}{\sqrt{\frac{7484}{225}} \times \sqrt{\frac{28154}{225}}} = \frac{11566}{\sqrt{210704536}} = \frac{11566}{14515.6652} = 0.7968$$

ii) **The linear regression has the form** $y = ax + b$ where

$a = \frac{cov(x,y)}{var(x)} = \frac{\frac{11566}{225}}{\frac{7484}{225}} = \frac{11566}{7484} = 1.5454$ and

$b = \bar{y} - a\bar{x} = \frac{464}{15} - 1.5454 \times \frac{151}{15} = \frac{464}{15} - \frac{233.3554}{15} = \frac{230.6446}{15}$

Therefore the regression line is $y = 1.5454x + 15.3763$

Prediction

ii) If x=45, then $y = 1.5454(45) + 15.3763 = 84.9193$

b) **Expected value of each decision**:

Model A: $EMV = 0.4(10{,}000) + 0.2(15{,}000) + 0.4(12{,}000) = \$11{,}800$

Model B: $EMV = 0.4(8{,}000) + 0.2(18{,}000) + 0.4(14{,}000) = \$12{,}400$

Model C: $EMV = 0.4(6{,}000) + 0.2(16{,}000) + 0.4(21{,}000) = \$14{,}000$

Choose the model with largest EV, Model C.

Expected Value of Perfect Information

$$EVPI = EPC - EMV$$

$$EVPI = 0.4(10{,}000) + 0.2(18{,}000) + 0.4(21{,}000) - 14{,}000 = \$2{,}000$$

If perfect information were available, the decision maker should be willing to pay up to $2,000 to acquire it.

QUESTION SEVEN

	Marking guide	Marks
a)	State five steps (1 Mark for each correct step)	5
	Maximum marks	**5**
b)	Formulation of objective	1
	Formulation of constraints	1
	Addition of slack variables	1
	Initial table	1
	Value of b	1
	Value of a	1
	Finding optimum solution	2
	Maximum marks	**8**
c)	Formulation of objective	1
	Formulation of supply constraints (0.5 Marks to each, max 2)	2
	Finding Initial Feasible Solution (1 Marks, max 3)	3
	Finding optimum solution	1
	Maximum marks	**7**
	Total Marks	**20**

Model Answer

a) Steps for crashing projects

Identify the Critical Path: Confirm that Activities B and C are indeed on the critical path. This ensures that any delay in these activities directly impacts the project timeline.

Calculate Crash Times: Determine how much each activity can be shortened (e.g., can Activity B be crashed by 1 day or 2 days, and similarly for Activity C?).

Cost Analysis: Assess the costs associated with crashing each activity. For example, crashing Activity B may cost $1,000 for 1 day, while crashing Activity C may cost $800 for 1 day.

Evaluate Dependencies: Consider any dependencies or constraints that might affect the crashing of these activities. Will crashing one activity impact others?

Select the Best Option: Compare the cost per day saved for each activity. Choose the activity with the least cost per day saved, provided it does not disrupt project dependencies.

b) Linear Programming

Let a and b represent the number of products A and B produced respectively

$Max\ P = 23a + 32b$ OBJECTIVE FUNCTION

Constraints

X; St $10a + 6b \leq 2500$

Y; $5a + 10b \leq 2000$ STRUCTURAL CONSTRAINTS

Z; $a + 2b \leq 500$

a ≥ 0 and b ≥ 0. NON-NEGATIVITY CONSTRAINT

Maximize $Z = 23\,a + 32\,b + 0S1 + 0S2 + 0S3$

S.T. $10\,a + 6\,b + 1S1 = 2500$

$$5\,a + 10\,b + 1S2 = 2000$$

$$1\,a + 2\,b + 1S3 = 500$$

and a, b, S1, S2 and S3 all ≥ 0.

In Simplex version, all variables must be available in all equations. Hence the Simplex format of the model is:

$$Maximise\ Z = 23\,a + 32\,b + 0S1 + 0S2 + 0S3$$

S.T.

$$10\,a + 6\,b + 1S1 + 0S2 + 0S3 = 2500$$

$$5\,a + 6\,b + 0S1 + 1S2 + 0S3 = 2000$$

$$1\,a + 2\,b + 0S1 + 0S2 + 1S3 = 500$$

and a, b, S1, S2 and S3 all ≥ 0.

Initial Table

Z	a	b	S1	S2	S3	RIGHT HAND SIDE	BASIC VARIABLE	RATIO
1	-23	-32	0	0	0	0		
0	10	6	1	0	0	2500	S1	2500/6
0	5	10	0	1	0	2000	S2	200
0	1	2	0	0	1	500	S3	250

Z	a	b	S1	S2	S3	RIGHT HAND SIDE	BASIC VARIABLE	RATIO
1	-7	0	0	3.2	0	6400		
0	-7	0	-1	0.6	0	-1300	S1	1300/7
0	0.5	1	0	0.1	0	200	b	
0	0	0	0	0.2	-1	-100	S3	

Z	a	b	S1	S2	S3	RIGHT HAND SIDE	BASIC VARIABLE
1	0	0	1	3.8	0	7700	Z
0	1	0	1/7	0.6/7	0	1300/7	a
0	0	1	-1/7	0.8/7	0	1500/7	b
0	0	0	0	0.2	-1	-100	S3

As all the elements of net evaluation row are either negative elements or zeros, the solution is optimal. Also the profit earned is equal to the shadow price. The answer is the company has to manufacture: 185.7 units of A and 107.14 units of B and the optimal return is Z = FRW 7,700

c)

$$Max\ profit = 4\boldsymbol{X_{11}} + \boldsymbol{5X_{12}} + \boldsymbol{6X_{13}} + \boldsymbol{3X_{14}} + \boldsymbol{3X_{21}} + \boldsymbol{6X_{22}} + \boldsymbol{4X_{23}} + \boldsymbol{5X_{24}} + \boldsymbol{2X_{31}} + \boldsymbol{X_{32}} + \boldsymbol{3X_{33}} + \boldsymbol{4X_{34}}$$

Demand constraint

$$st\ \boldsymbol{X_{11} + X_{12} + X_{13} + X_{14} \leq 10}$$

$$\boldsymbol{X_{21} + X_{22} + X_{23} + X_{24} \leq 12}$$

$$X_{31} + X_{32} + X_{33} + X_{34} \leq 7$$

Supply constraint

$$X_{11} + X_{21} + X_{31} \geq 10$$
$$X_{12} + X_{22} + X_{32} \geq 8$$
$$X_{13} + X_{23} + X_{33} \geq 5$$
$$X_{14} + X_{24} + X_{34} \geq 6$$

	A	B	C	D	Capacity/supply
X	4	5 1	6 3	3 6	**10 4 0**
Y	3 10	6	4 2	5	**12 2 0**
Z	2	1 7	3	4	**7 0**
Requirement/demand	**10 0**	**8 1 0**	**5 3**	**6 0**	

Initial Feasible Solution; $X_{12} = 1,\ X_{13} = 3,\ X_{14} = 6,\ Y_{21} = 10\ Y_{23} = 2,\ Z_{32} = 10$

Maximum profit

$$P = 5(1) + 6(3) + 3(6) + 3(10) + 4(2) + 1(7) = 5 + 18 + 18 + 30 + 8 + 7 = 86$$

End of Marking Guide & Model Answers